SYMMETRICAL 2N-PORT DIRECTIONAL COUPLERS

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Abstract

This is a theoretical study of 2N-port symmetrical directional couplers, with particular emphasis on six-port devices. Four classes are treated, with design principles given. One application is a power divider for use in a steerable 3-element phased array antenna.

I. Introduction

The four-port directional coupler is a well known microwave network. Most forms involve two parallel transmission lines or waveguides which are inter-coupled at discrete intervals along their length, or continuously. The principle of directional coupling can be extended to 3 or more parallel transmission lines. This has been treated by a few authors, particularly Kelleher [3] [4], and Boyd [1], who treated branch-line devices analogous to that of Fig. 3.

Fig. 1 shows, in block form, a 6-port version, together with the ideal scattering matrix for a forward coupler. This device has end-to-end symmetry and 3-fold axial symmetry in 3 angles. The general treatment for 2N ports involves similar N-fold angular symmetry with all lines lying on a circle about the axis.

II. Normal-Mode Analytic Technique

This method is extensively used to analyze 4-port devices, and one version was used for the 2N-port case by Boyd. A "normal mode" for this case is a set of N simultaneous equal-frequency excitations at the N ports on one end, so chosen in amplitude and phase that the set propagates from end to end on each line with a common transmission coefficient and a common reflection coefficient. For the symmetrical case of Fig. 1, there are N such normal modes, all involving equal amplitude excitations, but, with various phases as given by the equation

$$V_{n,m}/V_{1,m} = \exp\left[-j2\pi(n-1)(m-1)/N\right] \qquad (1)$$

where $V_{n,m}$ is the wave vector applied at the nth port in the mth normal mode, and $V_{1,m}$ is the input vector at port 1. For the first mode ($m = 1$) the excitations are in phase. For the other modes the phases are retarded by equal increments counting around the circle, but with increments different for each mode.

We wish to determine the scattering matrix for the complete device, S_{ik}, which involves 2N x 2N terms. Because of symmetry, only the first column need be derived, the others having the same elements suitably transposed. These terms are the outputs at all 2N ports when zero-phase exitation of 1 volt is applied to port 1, including the reflection term S_{11}.

For each normal mode, there is, by definition, only one reflection coefficient Γ_m and one transmission coefficient T_m, $m = 1, 2, \cdots N$. Knowing these, the scattering matrix S_{ik} can be directly determined. If all N modes are simultaneously applied such that each has 1/N volts and zero phase at port 1, then their sum is 1 volt at port 1 and zero at all the remaining input ports. The first column of S_{ik}, is,

For $n = 1, 2, \cdots N$

$$S_{n1} = \frac{1}{N}\sum_{m=1}^{N}\Gamma_m \exp\left[-j2\pi(m-1)(n-1)/N\right]$$

For $n = (N+1), (N+2), \cdots 2N$:

$$S_{n1} = \frac{1}{N}\sum_{m=1}^{N} T_m \exp\left[-j2\pi(m-1)(n-N-1)/N\right] \qquad (2)$$

III. Distributed Coupling Device

Fig. 2 shows a 6-port device using distributed coupling, analogous to the familiar 4-port backward-wave coupler. The three normal modes are (1) in phase; (2) phases of 0°, -120°, -240°; and (3) 0°, $+120^\circ$ and $+240^\circ$. This line is assumed to have a homogeneous dielectric and to propagate TEM modes only. For each mode, the phase velocities are equal, but, different modal characteristic impedances are involved, Z_1, Z_2 and Z_3, where $Z = V/I$ where I is the current in one line, and V is the voltage from one line to the shield. Each line is connected to an external line of impedance Z_o. Symmetry requires $Z_2 = Z_3$. For an electrical length θ, the coefficients Γ_m and T_m are easily shown to be

$$T_m = 2/\left[2\cos\theta + j\sin\theta\,(Z_m/Z_o + Z_o/Z_m)\right] \qquad (3)$$

$$\Gamma_m = \frac{-j\sin\theta\,(Z_o/Z_m - Z_m/Z_o)}{2\cos\theta + j\sin\theta\,(Z_o/Z_m + Z_m/Z_o)} \qquad (4)$$

This device is a reverse coupler, $S_{15} = S_{16} = 0$. From (2) it can be shown that this requires that $T_1 = T_2 = T_3$. From (3), this is met if

$$Z_1/Z_o = Z_o/Z_2 = Z_o/Z_3 \qquad (5)$$

Applying (2) we obtain the scattering parameters below, if (5) is met.

$$S_{11} = \frac{-j\sin\theta}{3}\,\frac{(Z_1/Z_o - Z_o/Z_1)}{2\cos\theta + j\sin\theta\,(Z_1/Z_o + Z_o/Z_1)}$$

$$S_{21} = S_{31} = -2\,S_{11}$$

$$S_{41} = 2/\left[2\cos\theta + j\sin\theta\,(Z_o/Z_1 + Z_1/Z_o)\right]$$

$$S_{51} = S_{61} = 0. \qquad (6)$$

It will be noted that S_{11} does not vanish. This device cannot be impedance-matched at all ports simultaneously. It can be matched at two ports, one at either end, by adding transformers, but, this modifies the characteristics for exitation at other ports.

IV. The Branch Line Coupler

Fig. 3 shows a 6-port device which is a forward coupler. Here, only the results of the analysis will be given, using the method of section II. The device uses external lines of impedance Z_o, "through" lines of electrical length θ_t and impedance Z_t, and "branch" lines of length θ_b and impedance Z_b. For a forward coupler, $S_{11} = S_{21} = S_{31} = 0$. This is achievable if $\Gamma_1 = \Gamma_2 = \Gamma_3 = 0$, which will occur for $\theta_t = 90^o$, $\theta_b = 75.52^o$, and

$$Z_b/Z_o = (12/5)\Big/(Z_o^2/Z_t^2 - 1) \qquad (7)$$

The scattering parameters, at band center, are:

$$S_{11} = S_{21} = S_{31} = 0$$

$$S_{41} = (1/3)\sqrt{1 - Z_t^2/Z_o^2} - jZ_t/Z_o$$

$$S_{51} = S_{61} = -\,2/3\sqrt{1 - Z_t^2/Z_o^2} \qquad (8)$$

Figure 4 shows computations for the frequency response of a 6-port branch-line device, giving 7.78 dB coupling for a power split 2/3, 1/6, 1/6.

V. A Lumped Element L-C 6-port Coupler

A number of lumped-element versions of the 4-port coupler are known. Fig. 5 shows a 6-port version. This is also a forward coupler. The design equations are

$$B_g = (1/X)\left(1 - \sqrt{1 - X^2/Z_o^2}\right) \qquad (9)$$

$$B_c = (2/3X)\sqrt{1 - X^2/Z_o^2} \qquad (10)$$

and, for band center, the scattering parameters are

$$S_{11} = S_{21} = S_{31} = 0$$

$$S_{41} = -(1/3)\sqrt{1 - X^2/Z_o^2} - jX/Z_o \qquad (11)$$

$$S_{51} = S_{61} = (2/3)\sqrt{1 - X^2/Z_o^2}$$

VI. Direct-Coupling through Multiple Irises

Fig. 6 shows one method by which three waveguides can be periodically intercoupled with many closely-spaced irises. Quantitative theory has not been developed here. If the irises have small individual effects, reverse coupling will be small due to destructive interference if their spacing is substantially less than $\lambda/2$. For the three normal modes, it is evident that the first (unison) mode will have a phase velocity which differs from the other two modes, which will be equal in phase velocity. If the propagation constants are $j\gamma_1$, $j\gamma_2$, and $j\gamma_3$, where $\gamma_2 = \gamma_3 = \gamma_1 + \gamma_d$, then

$$T_1 = \exp[-j\gamma_1 z]$$
$$T_2 = \exp[-j(\gamma_1 + \gamma_d) z]$$
$$T_3 = \exp[-j(\gamma_1 + \gamma_d) z] \qquad (12)$$

where z is the length, so that

$$S_{11} \simeq S_{21} = S_{31} \simeq 0$$

$$S_{41} \simeq 1/3\,[1 + 2\exp(-j\gamma_d z)]\,\exp(-j\gamma_1 z)$$

$$S_{51} = S_{61} \simeq 1/3\,[1 - \exp.(-j\gamma_d z)]\,\exp(-j\gamma_1 z) \qquad (13)$$

Depending upon γ_d and the length, it can be shown that the power split is given by

$$P_1 = P_{in}\left[5/9 + (4/9)\cos(\gamma_d z)\right]$$
$$P_2 = P_3 = P_{in}\left[2/9 - (2/9)\cos(\gamma_d z)\right] \qquad (14)$$

Maximum coupling occurs for $\gamma_d z = \pi$, at which point 4/9 of the power is found in lines 2 and 3 and 1/9 remains in line 1.

VII. A Phased Array Application

With three vertical antenna elements equilaterally spaced in a horizontal plane, directive characteristics are predicted theoretically [5] [6] as shown in Fig. 8. Using the switch and coupler arrangement of Fig. 7, four patterns are obtainable by switching. For one of the 3 directive patterns, the full signal is applied to one of the three coupler ports, the choice determining the direction. For an omnidirectional pattern, the power is equally split in the Wilkinson divider and applied in-phase to all three ports. The coupler of Fig. 3 and 4 provides the proper power split and phasing in each case.

References:

1 C. R. Boyd, IEEE Trans.MTT, 10, pp 278-294, July 1962.
2 E. Ott, IEEE Trans.MTT, 14, pp 578-579, Nov. 1966.
3 J. P. Shelton, IRE Conv.Record, Pt 1, 1957, pp 254-262.
4 J. P. Shelton and K. S. Kelleher, IRE Trans. AP, pp 154-161, Mar. 1961.
5 D. W. Atchley, et al, QST, Apr. 1976.
6 D. W. Atchley, et al, IEEE 26th Vehicular Tech. Conf., Mar. 1976.

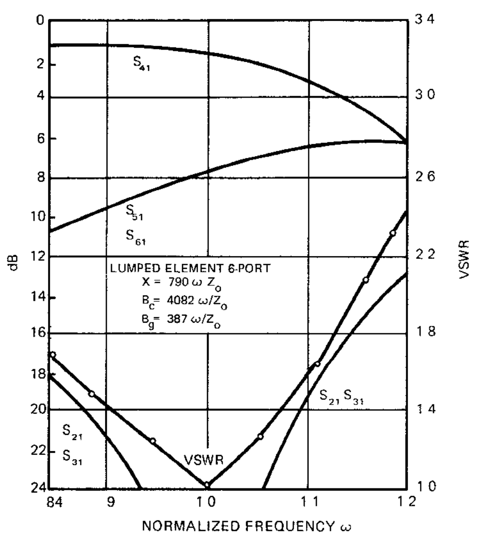
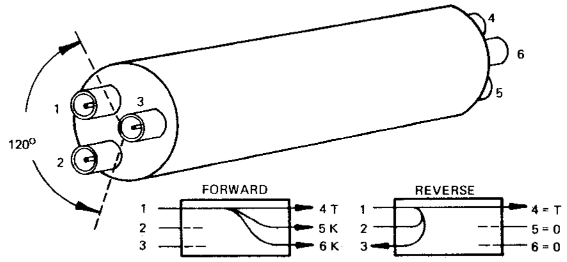


FIGURE 9. LUMPED ELEMENT 6-PORT FREQUENCY RESPONSE FOR 7.78 dB COUPLING AT BAND CENTER



OUT \ IN	1	2	3	4	5	6
1	0	0	0	T	K	K
2	0	0	0	K	T	K
3	0	0	0	K	K	T
4	T	K	K	0	0	0
5	K	T	K	0	0	0
6	K	K	T	0	0	0

FIGURE 1 SIX-PORT COUPLER & IDEAL SCATTERING MATRIX

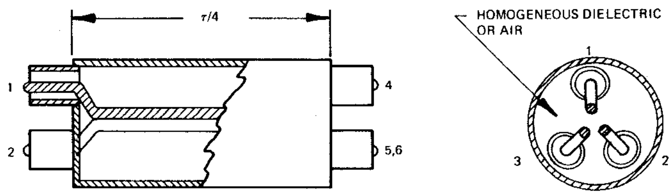


FIGURE 2 TEM REVERSE COUPLER

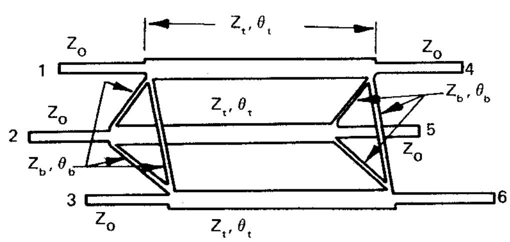


FIGURE 3 BRANCH LINE COUPLER

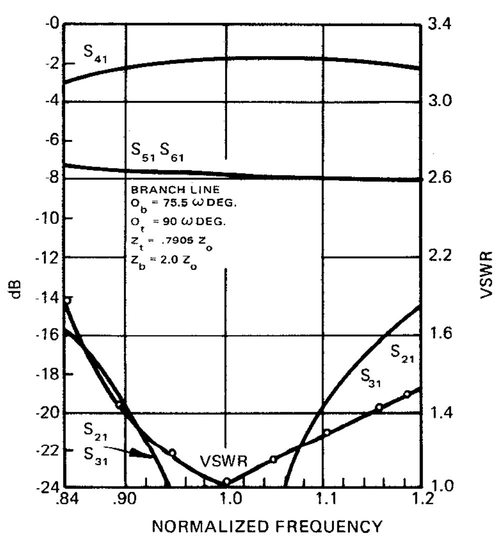


FIGURE 4 RESPONSE OF BRANCH-LINE DEVICE FOR 7.78 dB COUPLING

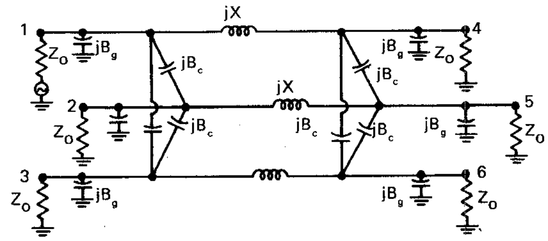


FIGURE 5 L-C 6-PORT COUPLER

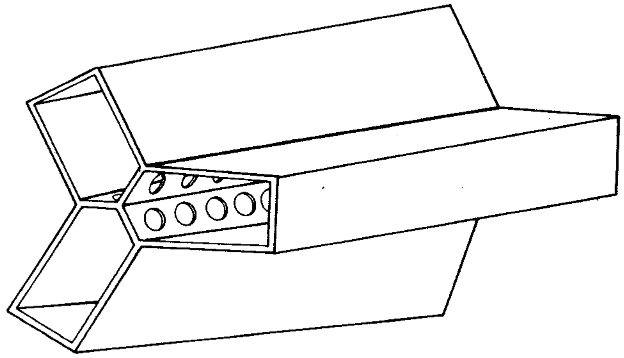


FIGURE 6 MULTIPLY COUPLED WAVEGUIDE TYPE

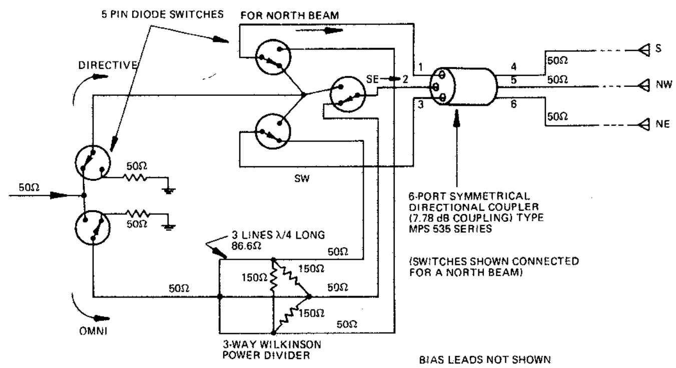


FIGURE 7 APPLICATION TO A SWITCHABLE PHASED ARRAY ANTENNA

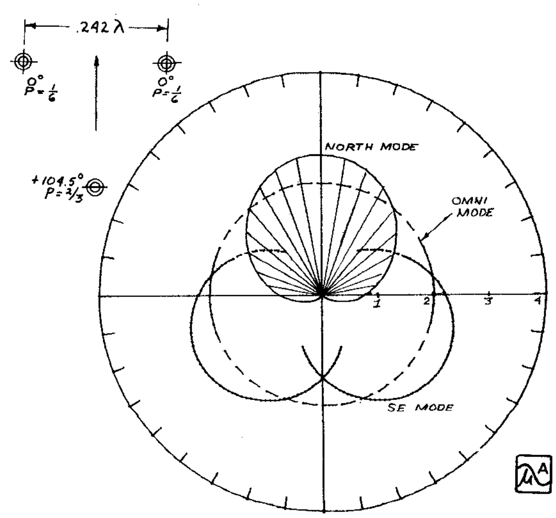


FIGURE 8 THEORETICAL BEAM PATTERNS, POWER DENSITY COMPARED TO A SINGLE ELEMENT

NOTES